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Class 12

Sub-.Maths

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Find the value of k in each of the Exercises 11 to 14 so that the function f is continuous at the indicated point:

11.

$$f(x) = \begin{cases} 3x - 8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases} \quad \text{at } x = 5$$

Solution:

Finding the left hand and right hand limits for the given function, we have

$$\begin{aligned} \lim_{x \rightarrow 5^-} f(x) &= 3x - 8 \\ &= \lim_{h \rightarrow 0} 3(5 - h) - 8 = 15 - 8 = 7 \end{aligned}$$

$$\lim_{x \rightarrow 5^+} f(x) = 2k$$

As the function is continuous at $x = 5$

$$\lim_{x \rightarrow 5^-} f(x) = \lim_{x \rightarrow 5^+} f(x)$$

So,

$$7 = 2k$$

$$k = 7/2 = 3.5$$

Therefore, the value of k is 3.5

12.

$$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & \text{if } x \neq 2 \\ k, & \text{if } x = 2 \end{cases} \quad \text{at } x = 2$$

Solution:

The given function $f(x)$ can be rewritten as,

$$f(x) = \frac{2^{x+2} - 16}{4^x - 16} = \frac{2^2 \cdot 2^x - 16}{(2^x)^2 - (4)^2} = \frac{4(2^x - 4)}{(2^x - 4)(2^x + 4)}$$

$$f(x) = \frac{4}{2^x + 4}$$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{h \rightarrow 0} \frac{4}{2^{2-h} + 4} = \frac{4}{2^2 + 4} = \frac{4}{4 + 4} = \frac{4}{8} = \frac{1}{2}$$

$$\lim_{x \rightarrow 2} f(x) = k$$

As the function is continuous at $x = 2$.

$$\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} f(x)$$

So, $k = \frac{1}{2}$

Therefore, the value of k is $\frac{1}{2}$

13.

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x \leq 1 \end{cases} \quad \text{at } x = 0$$

Solution:

Finding the left hand and right hand limits for the given function, we have